Erosive Burning of Solid Propellants

Erosive burning is the dependence of the burning rate of solid propellants on the crossflow properties of the burned products over the burning surface. This is in addition to the burning rate being dependent on the static pressure experienced by the surface. Let us call the latter burning rate as the one at zero crossflow or "normal" burning rate. There are quite a few burning rate equations proposed for the normal burning rate. The most widely used one is due to Saint-Robert and is given by,

$$\dot{r}_0 = ap^n \tag{1}$$

where a is the pre-exponent factor, n is the combustion index, and p is the static pressure experienced by the burning surface.

The most famous and widely recognized erosive burning model was developed by Lenoir and Robillard based on heat transfer theory [Lenoir, J. M. and Robillard, G., "A Mathematical Model to Predict the Effects of Erosive Burning of Solid Propellant Rockets," *Proceedings of the Sixth International Symposium on Combustion*, 1957, pp. 663-672.]. In this model they proposed the following mechanism. To maintain combustion, the solid propellant receives heat from two sources to bring each succeeding layer of propellant to the burning surface temperature T_s from the base temperature T_i.

The first source of heat is from the primary burning zone. The mechanism of heat transfer from this primary zone to the propellant is by a complex combination of conduction, heterogeneous turbulent convection, and radiation. The narrower the primary burning zone, the less resistance exists to heat transfer by this complex mechanism. Increased static pressure is considered to narrow the primary burning zone through an increase in the gas phase reaction rate. This mechanism of heat transfer rate is thus static-pressure dependent but it is independent of the crossflow velocity.

The second source of heat is from the crossflow of combustion products through the convective heat transfer and is dependent upon combustion flow rates. Thus the burning rate is proposed to be the sum of the two effects, a rate dependent on static pressure $\dot{r}_0$ and an erosive rate dependent upon the combustion-products crossflow rate $\dot{r}_e$. Thus,

$$\dot{r} = \dot{r}_0 + \dot{r}_e \tag{2}$$

where $\dot{r}_0$ is the normal burning rate component and $\dot{r}_e$ is the erosive burning rate component. The erosive burning rate component is postulated to be proportional to the convective heat transfer coefficient h under the condition of blowing and can be written with respect to convective heat transfer coefficient with zero blowing h_0 as

$$h = h_0 e^{\frac{-\beta \dot{r} \rho_p}{G}} \tag{3}$$

Substituting Eq. (3) into Eq. (2), we get,

$$\dot{r} = ap^n + kh = ap^n + kh_0 e^{\frac{-\beta \dot{r} \rho_p}{G}} \tag{4}$$

The convective heat transfer coefficient under zero blowing h_0 is correlated by Chilton-Colborn equation for flow over a flat plate,

$$h_0 = 0.0288 G c_p \, Re^{-0.2} \, Pr^{-0.667} \tag{5}$$

Although this equation is originally proposed for flow over flat plate, it can be applied to flow through grain ports by incorporating the characteristic dimension as the hydraulic diameter D. Combining Eqs. (5) and (4),

$$\dot{r} = ap^n + 0.0288Gc_p Re^{-0.2} Pr^{-0.667} ke^{\frac{-\beta\dot{r}\rho_p}{G}} \tag{6}$$

Noting $G = \rho u$ and $Re = uD\rho/\mu$, Eq. (6) is simplified to,

$$\dot{r} = ap^n + \frac{\alpha G^{0.8}}{D^{0.2} e^{\frac{\beta\dot{r}\rho_p}{G}}} \tag{7}$$

where,

$$\alpha = 0.0288c_p \mu^{0.2} Pr^{-0.667} k \tag{8}$$

An expression for k is derived by considering the energy balance between the heat transfer from the flame to the propellant surface and the heat required to raise the propellant temperature from its initial temperature T_i to the surface temperature T_s. The heat balance per unit area is given by,

$$h(T_0 - T_s) = \dot{r}_e \rho_p c_s (T_s - T_i) \tag{9}$$

This equation assumes that there is no significant exothermic or endothermic process occurring in the solid phase during the heating from T_i to the burning surface temperature T_s. Solving Eq. (9) and comparing with the earlier expression for $\dot{r}_e$ in Eq. (4),

$$\dot{r}_e = \frac{h}{\rho_p c_s}\left(\frac{T_0 - T_s}{T_s - T_i}\right) = kh \tag{10}$$

$$k = \frac{1}{\rho_p c_s}\left(\frac{T_0 - T_s}{T_s - T_i}\right) \tag{11}$$

Therefore, the erosive burning rate equation due to Lenoir and Robillard can be written as

$$\dot{r} = ap^n + \frac{\alpha G^{0.8}}{D^{0.2} e^{\frac{\beta\dot{r}\rho_p}{G}}} \tag{12}$$

where

$$\alpha = \frac{0.0288c_p \mu^{0.2} Pr^{-0.667}}{\rho_p c_s}\left(\frac{T_0 - T_s}{T_s - T_i}\right) \tag{13}$$

Although the value of β was proposed to be 53 by Lenoir and Robillard based on their experiments, the value of β can be chosen based on the experimental results of the motor and propellant under investigation.

Example

An aluminized composite propellant has the following properties.

Specific heat of solid propellant c_s	= 1400J/kg-K
Density ρ_p	=1750 kg/m^3
Pre-exponent factor a in the burning rate equation $\dot{r}_0 = ap^n$	= 3x10^{-5} m/s
Burning rate index n	= 0.4
Adiabatic flame temperature (stagnation temperature) T_0	=3610 K
Stagnation pressure p_0	= 7 MPa
Molar mass	=29.7 kg/kg mol
Specific heat at constant pressure of combustion products c_p	= 1975 J/kg-K
Viscosity of combustion products μ	=1.0049x10^{-4}Poise
Prandtl number Pr	=0.4922
Average surface temperature of burning propellant T_s	=1000 K
Propellant base temperature T_i	=300K

The hydraulic diameter of the grain port is 0.1m. If the propellant is assumed to follow the Lenoir-Robillard erosive burning rate model, calculate the total burning rate of the propellant for two crossflow Mach numbers of 0.5 and 0.6 at the given stagnation temperature. Distinguish the normal and erosive component of the burning rates. Assume that β in the Lenoir-Robillard equation to be 60. The Lenoir-Robillard equation is given by

$$\dot{r} = ap^n + \frac{\alpha G^{0.8}}{D^{0.2} e^{\frac{\beta \dot{r} \rho_p}{G}}}$$

where

$$\alpha = \frac{0.0288 c_p \mu^{0.2} Pr^{-0.667}}{\rho_p c_s}\left(\frac{T_0 - T_s}{T_s - T_i}\right)$$

Solution The total burning rate (normal burning component plus the erosive burning component) has be calculated for a fixed stagnation pressure under two crossflow Mach numbers. From the given values, the ratio of specific heats γ, and static pressures and mass fluxes for the two crossflow Mach numbers have to be calculated. Since the total burning rate $\dot{r}$ is implicit, the total burning rate has to be calculated through a suitable iteration.

Assumptions Although it is known that propellant surface temperature increases as the static pressure increases, its variation is small in the rocket operating pressure variation during equilibrium operation. Therefore, here the propellant surface temperature is assumed constant.

Analysis
The mass flux G is given by

$$G = \rho u = \frac{p}{RT} u$$

By routine gas-dynamic manipulations we get

$$G = M p_0 \sqrt{\frac{\gamma}{RT_0}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

$$\gamma = \frac{c_p}{c_p - R} = \frac{1975}{1975 - 8314.3/29.7} = 1.1652$$

$$G_{M=0.5} = 0.5\times 7\times 10^6 \sqrt{\frac{1.1652}{\frac{8314.3}{29.7}\times 3610}}\left(1+\frac{0.1652}{2}\times 0.25\right)^{\frac{-2.1652}{2\times 0.1652}} = 3287.06\,\frac{kg}{m^2 s}$$

$$G_{M=0.7} = 0.7\times 7\times 10^6 \sqrt{\frac{1.1652}{\frac{8314.3}{29.7}\times 3610}}\left(1+\frac{0.1652}{2}\times 0.49\right)^{\frac{-2.1652}{2\times 0.1652}} = 4056.834\,\frac{kg}{m^2 s}$$

$$\alpha = \frac{0.0288\times 1975\times\left(1.0049\times 10^{-4}\right)^{0.2} 0.4922^{-0.667}}{1750\times 1400}\times\left(\frac{3610-1000}{1000-300}\right) = 2.20344\times 10^{-5}$$

$$p = p_0\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$p_{M=0.5} = 7\times 10^6\times\left(1+\frac{0.1652}{2}\times 0.25\right)^{\frac{-1.1652}{0.1652}} = 6060205Pa$$

$$p_{M=0.7} = 7\times 10^6\times\left(1+\frac{0.1652}{2}\times 0.49\right)^{\frac{-1.1652}{0.1652}} = 5291290Pa$$

The normal burning rates are given by,

$$\dot{r}_{0_{M=0.5}} = 3\times 10^{-5}\times 6060205^{0.4} = 0.01549\,\frac{m}{s}$$

$$\dot{r}_{0_{M=0.7}} = 3\times 10^{-5}\times 5291290^{0.4} = 0.01467\,\frac{m}{s}$$

For the crossflow Mach number of 0.5, the total burning rate is given by,

$$\dot{r}_{M=0.5} = 0.01549 + \frac{2.20344\times 10^{-5}\times 3287.06^{0.8}}{0.1^{0.2} e^{\frac{60\times 1750\dot{r}_{M=0.5}}{3287.06}}} = 0.01549 + 0.022727\times e^{-31.9434\dot{r}_{M=0.5}}$$

Solving iteratively we get the total burning rates at the crossflow Mach number = 0.5 as,

$$\dot{r}_{M=0.5} = 0.025540\,\frac{m}{s}$$

The erosive burning rate component at the crossflow Mach number of 0.5 is,

$$\dot{r}_{eM=0.5} = \dot{r}_{M=0.5} - \dot{r}_{0_{M=0.5}} = 0.025540 - 0.01549 = 0.010051\,\frac{m}{s}$$

The erosive burning ratio ε, defined as the ratio of the total burning rate and normal burning rate, for the crossflow Mach number of 0.5 is given by

$$\varepsilon_{M=0.5} = \frac{0.025540}{0.01549} = 1.6488$$

For the crossflow Mach number of 0.7, the total burning rate is given by,

$$\dot{r}_{M=0.7} = 0.01467 + \frac{2.20344 \times 10^{-5} \times 4056.834^{0.8}}{0.1^{0.2}\, e^{\frac{60 \times 1750\, \dot{r}_{M=0.7}}{4056..834}}} = 0.01467 + 0.026894 \times e^{-25.8823\dot{r}_{M=0.7}}$$

Solving iteratively ε we get the total burning rate at the crossflow Mach number = 0.7 as,

$$\dot{r}_{M=0.7} = 0.027775 \frac{m}{s}$$

The erosive burning rate component at the crossflow Mach number of 0.7 is

$$\dot{r}_{e\,M=0.7} = \dot{r}_{M=0.7} - \dot{r}_{0_{M=0.7}} = 0.027775 - 0.01467 = 0.013105 \frac{m}{s}$$

The erosive burning ratio ε for the cossflow Mach number of 0.7 is given by

$$\varepsilon_{M=0.7} = \frac{0.027775}{0.01467} = 1.8933$$

Discussion As often said a successful model need not be of non-tractable mathematics; nor should it be fully correct. After Lenoir-Robillard model, although quite a few modelling efforts and improvements have been done for erosive burning effect, the model of Lenoir-Robillard captures the most observed behaviours of erosive burning phenomenon. Here in this example we find that the erosive burning effect is more for higher mass flux, which is a known fact. On further analysis you will find that the model predicts the erosive burning effect to be more for smaller motors (for smaller characteristic dimension D) and slower burning propellant, which are the observed behaviour in rocket motor operations.

The properties given in the example such as adiabatic flame temperature T_0, specific het of combustion products at constant pressure c_p, molar mass, viscosity, and Prandtl number can be determined for the chosen propellant by adopting standard codes such as CEC71.